

# Application of C-COM for Microwave Integrated-Circuit Modeling

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**Abstract** — The concurrent complementary operators method (C-COM) is extended for the FDTD simulation of microwave integrated circuits for the first time. Fields in the boundary layers are computed twice with the dispersive boundary condition (DBC) and its complementary operator to truncate the FDTD lattices. The two simulations are averaged to annihilate the first order reflections from the truncated boundary. Numerical error analysis show that the reflections are further suppressed by least 20dB due to the implementation of complementary operators, and the setup of parameters becomes easier and more robust. A flexible and highly efficient absorbing boundary condition for guided wave problems is thus obtained through the combination of C-COM and DBC. Simulation results for a modified microstrip transmission line and a microstrip impedance transformer are given to validate this method.

## I. INTRODUCTION

Recently, the complementary operators method (COM) was presented as a new FDTD boundary truncation scheme [1]. This method uses two complementary boundary operators whose reflection coefficients are identically opposite. The first order reflections from the truncated boundary are canceled by performing two independent simulations. In the concurrent version of this method, termed C-COM [2], the two complementary operators are employed concurrently in the same simulation, thus the computation is reduced about one half. In C-COM, the errors caused by the first order reflection for traveling waves and evanescent waves can be annihilated completely in any directions, and the errors will only be in the order of the second order reflections. If a high quality absorbing boundary condition (ABC) is used as the fundamental operator, the errors will be very small. Using Higdon's ABC as the fundamental operator, C-COM has been utilized to simulate the radiation of line sources and scattering from perfectly conducting cylinders [1][2]. It has been shown that the performance of this scheme is comparable with, or even superior to, that of the PML technique [3] for some applications.

In this paper, we will extend this method to multilayered microwave integrated circuits. In Higdon's ABC, the waves are assumed to travel with same speed and different incidence angles toward the boundary plane. For the

guided wave problems of interest to this work, we cannot directly use the algorithms in [1][2], since in Higdon's operator, there is the fundamental assumption of a uniform wave speed in the formulation. Based on the fact that when a Gaussian pulse travels in guided wave structures, the velocities of fields are different for different frequencies due to the dispersion property of these structures, we use the dispersive boundary condition (DBC) [4][5] developed to absorb waves over a reasonably wide frequency band, to configure complementary operators. The DBC can be considered as an extension of Higdon's ABC. The parameters in the DBC can be set to absorb waves with different velocities, and evanescent waves can also be absorbed. It has been shown that the DBC can obtain good absorption through optimizing the parameters [3]. In this paper, we will show that through the introduction of complementary operators, the performance of DBC will be further improved, and it will become more flexible to set these parameters.

The complementary operators for the DBC, and the concurrent implementation are presented briefly. The criteria for the selection of parameters are given, and the numerical error analysis is implemented to study the performance of the complemented DBC. The simulations of the modified microstrip transmission lines [6] and a microstrip transformer are used to validate the effectiveness of this method.

## II. COMPLEMENTED DISPERSIVE BOUNDARY CONDITION

The dispersive boundary condition is expressed as [4]

$$\left[ \prod_{i=1}^M \left( \frac{\partial}{\partial x} + \frac{\beta_i}{c} \frac{\partial}{\partial t} + \alpha_i \right) \right] \Phi = 0 \quad (1)$$

where  $c$  is the speed of light,  $\beta = \{\beta_1, \beta_2, \dots, \beta_M\}$  and  $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_M\}$  are parameters to be set. Its theoretical reflection coefficient is given by

$$\Gamma_M = - \prod_{i=1}^M \frac{-j\gamma_x + j\beta_i k + \alpha_i}{j\gamma_x + j\beta_i k + \alpha_i} \quad (2)$$

where  $\gamma_x$  is the complex wavenumber of the incidence wave.

Since the complementary operators can only annihilate the first order reflections, higher order operators should be considered to reduce the second order reflections. However, numerical stability considerations limit the application of high order operators. We use the third order dispersive operator represented by  $DBC_3$ , which can be obtained from (1) when  $M=3$ , as the fundamental operator to get the complementary operators. Suppose  $M=4$  and  $\alpha_4=0$  in (1), we have

$$\begin{cases} B_{4+}(\Phi) = \left[ \frac{\partial}{\partial x} DBC_3 \right] \Phi = 0, \quad \beta_4 \Rightarrow 0 \\ B_{4-}(\Phi) = \left[ \frac{\partial}{\partial t} DBC_3 \right] \Phi = 0, \quad \beta_4 \Rightarrow \infty \end{cases} \quad (3)$$

And the corresponding reflection coefficients are

$$\Gamma_{DBC_3} = \Gamma_{B_{4-}} = -\Gamma_{B_{4+}} \quad (4)$$

which indicates that  $B_{4+}$  and  $B_{4-}$  given in (3) are complementary. Since  $B_{4+}$  and  $B_{4-}$  have the same reflection coefficients as  $DBC_3$  in magnitude, if  $B_{4+}$  or  $B_{4-}$  is used as ABC independently, there are no improvements compared with  $DBC_3$ . But the reflection will be suppressed greatly as  $B_{4+}$  and  $B_{4-}$  are used as complementary operators.

In order to implement the complementary operators concurrently [2], the computation domain is divided into working domain and boundary domain. Within the working domain, each of the field components is computed once by FDTD formulas. But within the boundary domain, each of the field components is assigned two storage locations and each set of field components are updated independently using  $B_{4+}$  or  $B_{4-}$  as ABC. The values of the field components on the interface between working domain and boundary domain, which will be used to update the field components in working domain at next step, are obtained by averaging the two sets of values of field components obtained in the boundary domain.

In the boundary condition,  $\bar{\beta}$  is used to approximate the velocities of the guided waves. If  $\gamma_x = k_x - j\alpha_x$  is the complex wavenumber of the incidence waves, then

$$\Gamma_1 = \frac{j(k_x - \beta_1 k) + (\alpha_x - \alpha_1)}{j(k_x + \beta_1 k) + (\alpha_x + \alpha_1)} \quad (5)$$

if  $k_x / \beta_1 = k$  and  $\alpha_x = \alpha_1$ , then waves impinging on the boundary with wavenumber  $k_x$  and attenuation  $\alpha_x$  will be absorbed completely [4]. So  $\beta_1$  should be determined by the velocity of waves in the structures. An estimated  $\bar{\beta}$  for microwave planar circuits is

$$\bar{\beta} = \left\{ \beta_i \right|_{i=1,2,3} = \left\{ \sqrt{\epsilon_{\text{eff}}(f_i)} \right|_{i=1,2,3} \quad (6)$$

where  $\epsilon_{\text{eff}}$  is the effective dielectric constant, which is a function of frequency. And for a structure to be simulated,  $\epsilon_{\text{eff}}$  is unknown. But its exact value is not required. For dispersive boundary conditions, several techniques to set optimized value of  $\bar{\beta}$  have been studied [3]. As shown in [4], if higher-orders are used,  $\epsilon_{\text{eff}}$  can be selected in a wide range between the static solution and the exact dielectric constant of the structure. We have found that especially after complemented operators are used,  $\bar{\beta}$  can be selected in a wider range, and the results are robust. Hence another advantage of the application of the complementary operators is to relax the limits for the selection of parameters.

$\bar{\alpha}$  is used to simulate the attenuations of waves and stabilize the solutions since high order derivatives in the absorbing boundary condition are used.  $\bar{\alpha}$  can also be used to absorb the evanescent waves [4]. Our numerical experiments show, that as long as the solution is stable,  $\bar{\alpha}$  should be set as small as possible. Otherwise the accuracy of the low frequency response will be reduced. Fortunately we found that typical values such as  $\bar{\alpha} = \{0.05, 0.1, 0.15\}$  are applicable for many structures we have studied. An important requirement for the setup of  $\bar{\alpha}$  is that  $\bar{\alpha}$  must have at least one zero-element for truncation faces at the far and near ends of a transmission line. Otherwise reflections from these ends will become very large. This is because the attenuation of the principle propagation mode impinging on these two faces is very small ( $\alpha_x \approx 0.0$ ). As shown in equation (5), the values for  $\bar{\alpha}$  far from zero can result in very large reflections. Hence we always set  $\bar{\alpha} = \{0.0, 0.0, 0.0\}$  for truncation face at end of a transmission line, input or output port of a microwave component.

In order to study the effect of parameters in the boundary conditions, reflections for a shielded microstrip are studied. The reflection errors up to 20GHz for currents calculated by  $DBC_3$ ,  $B_{4+}$ ,  $B_{4-}$ , and C-COM with 10 boundary layers are given in Fig.1. The observation point is 10 cells away from the interface in C-COM and 20 cells away from the  $DBC_3$ ,  $B_{4+}$  and  $B_{4-}$  boundary. Since there

exist only propagating modes,  $\bar{\alpha}$  is set to zero. It is shown in Fig.1 that  $DBC_3$ ,  $B_{4+}$  and  $B_{4-}$  have almost same absorbing performances. Because the third-order is used and only ends need to be terminated, the absorbing performances are very excellent. The effective dielectric constant of this strip line is  $\epsilon_{refl} = 1.76$ , however, we can see that even if the parameters have large variation around this value, the absorbing performance is still very good for  $DBC_3$ ,  $B_{4+}$  and  $B_{4-}$  because of their high orders [4]. After using complementary operator, the effect of the change of parameters will be even smaller. So we can set the parameters  $\bar{\beta}$  around the square root of the static effective dielectric constant  $\epsilon_{refl}$  and can be varied up to the substrate dielectric constant  $\epsilon_r$ . 20dB improvement of accuracy through the use of complementary operators is observed as  $\bar{\beta}$  was changed over a wide range.

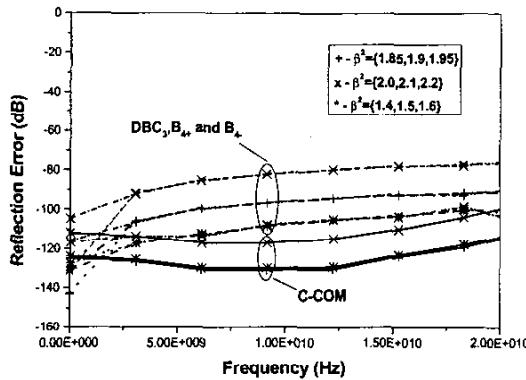


Fig. 1. Reflection error of the current on a shielded microstrip line with width 0.254mm, substrate height 0.254mm, and  $\epsilon_r = 2.2$ . The top of the PEC enclosure is at 0.762mm from the substrate, and the distance between the edges of the microstrip and the side PEC walls is 0.928mm.

A homogenous microstrip transmission line without shield is simulated by FDTD method with all boundaries except the ground plane truncated by C-COM and  $DBC_3$  respectively. The distances between the interface of C-COM, edges and surfaces of the microstrip are 10 cells, and 10 boundary layers are used. Equally in  $DBC_3$ , the distances between the faces of  $DBC_3$  and edges and surface of the microstrip are 20 cells. The observation point is only five cells away from the interface, or 15 cells away from the boundary truncated by  $DBC_3$ . Fig.2 demonstrates the effect of  $\bar{\beta}$  with fixed

$\bar{\alpha} = \{0.05, 0.1, 0.15\}$  for the top, left and right truncation faces.  $\bar{\alpha}$  is set to zero for faces at the near and far ends. The static effective dielectric constant of this microstrip line is about 6.58. With a series of  $\bar{\beta} = \{\beta_1, \beta_2, \beta_3\}$  with element values changed widely from  $\sqrt{5}$  to  $\sqrt{8.5}$ , the average reflections change was within 5dB. The improvement of accuracy through the use of complementary method is observed to be around 20 dB once again.

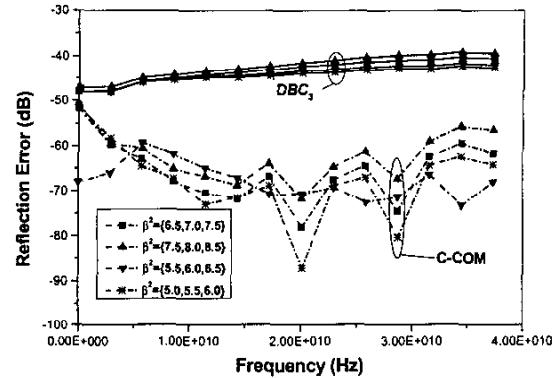


Fig. 2. Reflection error of the current on a microstrip line with width 0.264mm, substrate height 0.254mm, and  $\epsilon_r = 9.8$ .

### III. APPLICATION EXAMPLES

Fig. 3 shows the effective dielectric constant for a modified multilayered microstrip line that is developed for high quality RF/MMIC transmission lines and inductors [6]. The thickness of GaAs layer  $h = 75\mu\text{m}$ , the thickness of polyimide layer is  $d$ . The results are compared with data from [6]. In order to obtain better absorbing performance for different thickness, we have used different set of parameters as  $\bar{\beta} = \{\sqrt{7.5, 8.0, 8.5}, \sqrt{5.5, 6.0, 6.5}, \sqrt{3.5, 4.0, 4.5}, \sqrt{3.0, 3.5, 4.0}\}$  as the thickness changed. Two strategies can be used to define these parameters. The first one is based on using the prior knowledge of the transmission lines to be studied. The second one is to use some estimated values, such as the values between  $\sqrt{\epsilon_r \text{GaAs}}$  and  $\sqrt{\epsilon_r \text{ploy}}$ . Then the initial values of the effective dielectric constants are obtained. Based on these values, we reset the boundary conditions, and a better absorbing performance is obtained. The loss term is set as  $\bar{\alpha} = \{0.0, 0.0, 0.0\}$  for faces at the far and near ends and

$\bar{\alpha} = \{0.05, 0.1, 0.15\}$  for the top, left and right truncation faces as we have used in the previous computations.

Fig. 4 gives the return loss of a Chebyshev microstrip transformer compared to the Agilent-ADS MOMENTUM and SCHEMATIC solutions. This transformer is designed to transform the characteristic impedance from  $45\Omega$  to  $110\Omega$  with center frequency 20GHz. The static effective dielectric constants of the input and output ports are about 1.75 and 1.9, so we choose  $\bar{\beta} = \sqrt{[1.7, 1.9, 2.0]}$ . The loss term  $\bar{\alpha}$  is set same as for the computation in Fig. 3.

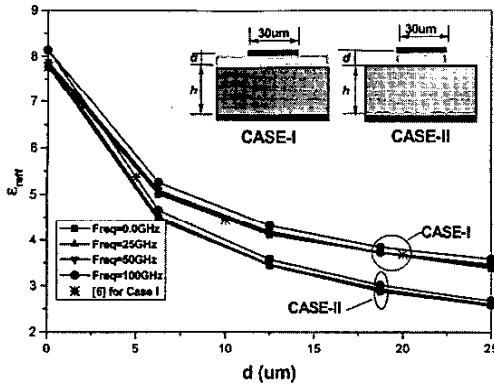


Fig.3. A modified high quality multilayered microstrip line for RFIC/MMIC. The width of the microstrip is  $30\mu m$ , and the substrate has two layers. The height of the GaAs layer is  $h=75\mu m$ ,  $\epsilon_{GaAs}=12.9$ . The dielectric constant for the polyimide layer is  $\epsilon_{poly}=3.2$ , its thickness  $d$  is changed from 0 to  $25\mu m$ . In case II, the surrounding polyimide is etched away.

#### IV. CONCLUSION

In this paper, we have successfully extended the C-COM [2] to multilayered microwave planar circuits in combination with dispersive boundary condition [4][5] for the first time. Error analysis shows that the reflection performance of dispersive boundary conditions can be improved by at least 20dB through the application of complementary operators. The implementation of complementary operators also makes the selection of parameters in the dispersive boundary condition more flexible. For all of the structures we have studied, the average reflection errors are smaller than  $-65dB$  without optimizing the parameters. We have used this method has been used to simulate a variety of guided wave structures such as the multilayered modified microstrip transmission lines and microwave circuits.

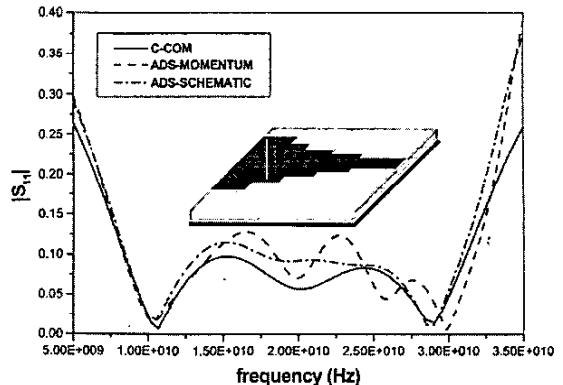


Fig.4. Return loss of a Chebyshev microstrip transformer. The results are compared with Agilent-ADS MOMENTUM and SCHEMATIC solutions. The dielectric constant of the substrate is 2.2, and thickness is  $0.794mm$ . The widths of the input and output ports are  $2.78mm$  and  $0.556mm$ , respectively. The lengths of the three quarter-wavelength sections are chosen to be  $2.735mm$  with designed center frequency 20GHz. Their widths are  $1.946mm$ ,  $1.39mm$  and  $0.843mm$ .

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